CABLE ELEMENT ANALYSIS TECHNIQUES

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ABSTRACT

The recent developments have taken place during the three decades in the cable-suspended and cable-stayed bridges that are very appealing aesthetically and are also very important structures. Slender masts and towers laterally supported by gays or stiff cables are usually adopted for this purpose as they provide economical solutions compared with freestanding towers. The behavior of such structures is influenced by many factors and parameters making a highly complex problem. This highly non-linear problem is exhibited either by gays due to sags, low level of pretension or by masts or towers due to large displacements and beam-column effects, in addition to the effect of wind loading at various heights, initial geometry effects such as initial tension in cables, initial imperfection in tower members, lack of fit, temperature, support yielding, etc. Thus, such structures experience simultaneous increase in axial thrust and lateral loads in both directions, such a combination can actually introduce a highly non-linear problem that could be solved either by finite element technique or by beam-column theory. In this paper the cable element used in the cable - frame interaction structures such as bridges and towers of all types and purposes shall be discussed from all aspects concerning the material, the arrangement and structural details.

طرق تحليل العناصر الوترية

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الخلاصة:

تعتبر المنشآت المعلقة من الظواهر الهندسية الحضارية لما لها من خصوصية هندسية وجمالية معمارية أخاذة, ونظراً لأصالتها التاريخية من جهة والتطورات الهندسية والتقنية التي طرأت على أسلوب تصميم وتحليل وتنفيذ هذه المنشآت لما لها من تعقيدات معروفة ناتجة من عدم خطيتها من جهة أخرى, فإن من الضروري التعرف على أهم هذه التطورات والأساليب الحديثة ومنها طريقة العناصر المحددة وكذلك طريقة العتبة- العمود. يقدم هذا البحث عرضاً شاملاً مدعماً بالأشكال والجداول مع ملحق لأهم وأحدث طرق تحليل العناصر الوترية (الحبال) والتي تستخدم في المنشآت والجسور المعلقة وأبراج الاتصالات ذات الارتفاعات الشاهقة.

Cable-structure interaction and methods of connection

I. Idealized interaction

In the analysis or design of cable-stayed towers or bridges, different combinations are possible between supports for the towers and cable attachment to them, as shown in table (1). The tower base generally is either fixed (as in short span cable-stayed and cable-suspended bridges, and some types of guy towers) or pin (as in most guy towers and long-span stay bridges).

The pin-connections are subjected to very heavy bearing stresses in addition to their complicated erection, rocker-towers, pin bearing at the base, afford the most economic and scientific design for bridges of longer span, they eliminate the stresses from unbalanced cable forces without requiring movable saddle construction, if rocker tower are adopted, they must be secured against tipping during erection. In addition this type of connection prevents the cable shortening owing to the 3-D effect caused by twisting moments induced by the lateral movement of the connected joints.

When fixed base is used then the tower is firmly anchored at the base of the tower pedestals, it is free to deflect longitudinally under the deformations of the cables when they are fixed to tower.

The cables are generally continuous over saddles located inside the towers, saddles are either bolted to the supports or provided with rollers, when fixed saddles are used, the resultant unbalanced horizontal forces must be calculated and allowed in the design of the tower, unless it is of the rocker type.

If the saddles are movable, the eccentricity of the vertical reaction under various loading conditions must be accounted for. The roller support for the saddle permits its horizontal movement. The resultant tensions of the cable should pass through the middle of the roller nest to give an even distribution of stress. The friction of the rollers is so small that the angle of inclination of the resultant reaction is negligible.

Instead of circular rollers, rockers may be used so as to furnish a greater diameter thereby reducing friction and roller bearing stress. Rockers, however, must be secured against excessive motion liable to cause overturning. Rollers, meanwhile serve to reduce the bending stresses in the towers due to an unbalanced horizontal cable pull resulting from special loading conditions and temperature. On the other hand, they are expensive, add erection complications, increase maintenance, and merely substitute eccentric vertical loading for the unbalanced horizontal pull. On, the whole, fixed saddles provide a simple and safer solution [22].

II. Actual interaction

The fixed and pin saddles described earlier give an idealized picture of the field fixed or movable cable supports, a typical arrangement of these supports differs from one structure to another, they are usually provided at the top of the tower as well as at intermediate locations along the tower, depending on the number of the cables used. These supports are either: -

- Fixed to the tower by means of welding or riveting.
- Movable supports by means of rocker or roller devices.

There are other effects that must be taken into considerations such as anchoraging cables to pedestals, cable numbers and cable spacing and arrangement.

The Guy Analysis:

This type of analysis depends on the behavior of inclined cable under its own weight -see figs.1 & 2-, which follows two approaches [2], [17]&[22]:

- The exact catenary approach.
- The approximate parabola approach.

When the required data of cables (such as dimensions, and initial tensions) for the certain structure (towers or bridges) are unknown, a type of analysis called the preliminary analysis must be used in order to evaluate the initial values of cable tensions in order to include it in the next stage of analysis of the structure which is the non-linear analysis.

The exact catenary approach

The differential equation of the equilibrium curve shown in figures (1) & (2) is [17]: -

$$\frac{d^2y}{dx^2} = \frac{-w\sec\varphi}{-H} \quad \text{------ (1)}$$

where:

φ is the inclination of the cable to the horizontal at any distance "x" since $\tan\phi = \frac{dy}{dx}$, so (eq.1) may be written as[(17)&(22)]:-

$$\frac{d^2y}{dx^2} = -\frac{w}{H}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} \quad \text{----(2)}$$

by integrating and organizing[(17)&(22)]:-

$$y = \frac{1}{2\frac{w}{H}}\left(e^{w/H-x} + e^{-w/H-x} - 2\right) \quad \text{----(3)}$$

This equation is called the catenary equation; therefore, a cable under its own weight hangs in a catenary. When the stay is taken to be a straight bar then the elastic stretch or elongation is [26]: -

$$\Delta L_s = \frac{(F_1 - F_0)}{EA\cos\varphi_0^2}\, l \quad \text{------ (4)}$$

The approximate parabolic approach

If the sag ratio η= f / l is small, all the formula for the catenary may be replaced, with sufficient accuracy, by the formula for parabolic cables, this means that the cable is subjected to uniform vertical load only. Although a parabola is only an approximation to the exact catenary analysis, the parabolic equation is familiar to most practicing engineers [(4) & (10)]. The elastic stretch or shortening in the cable is approximately equal to (see Figs. (5) & (6)): -

$$\Delta L_s = \frac{HL_c}{A\mathrm{E}\cos\varphi_o} \quad \text{----- (5)}$$

The preliminary analysis

The first step to non-linear analysis is to obtain an approximate set of reactions at the guy level [(2)&(3)]. The tower is assumed to behave as a continuous beam over rigid supports, with the supports located at each guy level. A standard linear analysis is utilized in which the tower segments between supports are treated as beam-column elements using the ordinary (4) and (2) stability functions [17] at ρ=0, so the tower is treated as an ordinary structure (with no cables) having either solid section or lattice section depending on the type of tower, the reactions of the rigid supports then represents the initial guy reactions which are ready to enter the non-linear analysis. The initial tensions are calculated in the cables by the following means [4]: -

- Measuring accurately the position of the end joints and control the sag of the cable.
- Controlling accurately the zero stress length and final length of the cable due to initial tensions.
- If cable is attached to eye bar links at the end initial measuring the stress in the eye bar can control tension.

The non-linear cable analysis

In spite of the fact that the behavior of the material of the structural elements in cable-stayed structures is linear-elastic, the overall load displacement relationship for the structure is non-linear under normal design loads [12] , the overall non-linear behavior originates from four primary sources[9], [10], [13],&[27]: -

1. The non-linear axial force-elongation relationship for the inclined cables stays due to sag caused by their own weight.
2. The non-linear axial force and bending movement deformation relation for the towers and masts (i.e. the stability non-linearity including the bowing effect).
3. The geometrical changes caused by large displacement under normal design loads.
4. Special effects increasing results of non-linearity (such as, temperature, industrial imperfections, support yields, …etc.) both in mast and cables [27].

The bowing effect cannot be taken into account in the cable elements since they cannot be treated as flexural elements or beam-column elements. This means that no flexural rigidity, though no application of stability or bowing functions could be concluded. The non-linear behavior of the cable is treated by many procedures but the most important one is the equivalent modulus of elasticity technique [6].

As it will be detailed in Appendix-A- the "E" value of the cable is a function of many variables (see eq. A-3, 4). When the cable tension increases, the sag decreases (see figure 4), and the apparent axial stiffness of the inclined cable increases. A convenient method to account for this variation in the cable axial stiffness is to consider an equivalent straight chord member "L" with an equivalent modulus of elasticity [7]. This combines both the effect of material and geometrical deformations such that the axial stiffness of the equivalent chord member becomes equal to the apparent axial stiffness of the actual curved cable, this concept was first introduced by ***Ernst***, and has been verified by several additional investigators [4]. This "Equivalent" of the cable is given by two approaches –see Fig.3- [6], &[7]:

a. The tangent "Eeq."... b. The secant "Eeq.".

$$\tan \mathrm{E}_{eq.} = \frac{\mathrm{E}_e}{1+\frac{(wl)^2 A\mathrm{E}_e}{12F_i}} \text{------- (6)}$$

Notice that this equation is another form of (eq. A- 3), while: -

$$\sec E_{eq.} = \frac{E_e}{1 + \frac{(wl)^2 (F_i + F_f) A E_e}{24 F_i^2 F_f^2}} \text{------- (7)}$$

Equation (6) gives the tangential or instantaneous value of the equivalent modulus when the tension in the cable equals "Fi" or when the cable tensile stress is equal to "σ", if the cable changes from "Fi" to "Ff" during the application of a certain load increment (which is equivalent to a change in cable stress from "σi" to "σf", then the secant value of the equivalent modulus of elasticity over the load increases [(17)].

In the non-linear analysis, all cables are treated initially as straight members and represented geometrically by their chords, the actual shape, however, is a catenary. It can be replaced by parabola to avoid tedious calculations and especially when sags are smaller or tensions are higher than true length of the curved cable (arc length "L") which can be expressed by a series [(12)]: -

$$L = L_c \left(1 + \frac{W^2 \cos^4 \varphi_o}{24 H^2} - \frac{W^2 \cos^4 \varphi_o}{640 H^4} + \ldots \right) \text{---------- (8)}$$

The third and subsequent terms inside the bracket are very small compared to the first and second terms, so they can be neglected, then the differentiation of (eq.8) yields [(12)]: -

$$\mathrm{ds} = \left(1 + \frac{W^2 \cos^4 \varphi_o}{24 H^2} \right) \Delta L - \left(\frac{L_c W^2 \cos^4 \varphi_o}{12 H^3} \right) \mathrm{dH} \text{---------- (9)}$$

Which can be transformed to the elongation of the cable due to the change in "H" [(12)]:-

$$\frac{ds}{dH} \approx \frac{L}{EA \cos \varphi_o} \text{-------- (10)}$$

Therefore, the axial deformation of the chord due to the change in "H" is [(12)]: -

$$\frac{dL_c}{dH} = \frac{L}{EA \cos \varphi_o} + \frac{L_c W^2 \cos^4 \varphi}{12 H^3} = \frac{H L_c}{AE \cos \varphi_o} \left(1 + \frac{w^2 l^2}{12 H^2} \right) \text{---------- (11)}$$

On the right hand side of this equation, the first term, is linear representation of the elastic stretch of the cable (i.e eq. 4 or 5), while the second term, is non-linear and inversely proportional to "H_3"- see fig. (4)&(5)-. If a parabola cable shape is adopted, a modified stiffness of cable should be used in the analysis, so [(12)]: -

$$EA)_{\text{mod.}} = r(AE) \text{---- (12-a)...where: } r = \frac{1}{1 + \frac{w^2 \cos^5 \varphi_o EA}{12 H^3}} \text{-------(12-b)}$$

However, "r" varies as "H" varies as "H" increases from "H_1" to "H_2", an average cable stiffness should be used so that [(12)]:

$$EA)_{mod.} = r_1(EA) \text{ -- (12-c)..where: } r_1 = \frac{1}{1+\dfrac{w^2\cos^5\varphi_o EA(H_1+H_2)}{12H_1^2H_2^2}} \text{ -- (12-d)}$$

Which means an iterational technique of solution must be adopted and that the cable stiffness is adjusted in every iteration, however, the non-linear effect can be compensated by applying a pair of imaginary forces "K" at the ends of each cable, instead of changing the cable matrices in each step. So, for the "hth" iteration, the condition of equilibrium gives [(12)]:

$$K = \frac{EA)_o - EA)_h}{EA)_o} F_{(n-1)} \text{ ------- (13)}$$

The second non-linear effect of cables is caused by the change of the angle "φ" due to displacement at both ends, this change in slope, or the rotation of the chord, is small compared with the original slope, therefore the change in vertical and horizontal components of the cable forces are approximately [(12)]: -

$$\begin{aligned} \Delta H &= -F\sin\varphi\Delta\varphi \\ \Delta V &= S\cos\varphi\Delta\varphi \end{aligned} \text{ ------ (14-a)}$$

$$\Delta\varphi = \frac{(u_G + w_T)\sin\varphi + (w_G - u_T)\cos\varphi}{L_c} \text{ ---------- (14-b)}$$

Summing up both non-linear effects of cables and the imaginary forces that have to be applied at the end of the cables after the "nth" iteration [(12)]: -

$$\begin{aligned} F_1 &= F_n\sin\varphi\Delta\varphi - K_n\cos\varphi \\ F_2 &= F_n\cos\varphi\Delta\varphi - K_n\sin\varphi \end{aligned} \text{ -------(15)}$$

The above detailed non-linear behavior can be replaced by the equivalent modulus of elasticity of cable (eq. 6-7) to include all the above geometrical non-linear effects in addition to any material with non-linear behavior, so introducing an additional effect of temperature [(12)]: -

$$\Delta L_{temp.} \approx \varphi t \times L = \varphi t \times L_c\left(1+\frac{w^2l^2\cos^2\varphi_o}{24H^2}\right) \text{---------- (16)}$$

And rearranging all the effects of parabolic equation we have what we called the cable non-linear equation [(12)]: -

$$f(H) = \frac{L_c}{AE_e}H^3 - H^2\left[\Delta L_c + \frac{H_oL_{co}}{AE_e\cos\varphi_o}\left(1+\frac{w_o^2l^2}{12H_o}\right) - \frac{L_{c_o}}{24}w_o^2l^2\frac{\cos^2\varphi_o}{H_o^2} - \varphi t_cL_c\right] +$$
$$H\frac{w^2l^2L_c}{12AE_e} - \frac{L_cw^2l^2\cos^2\varphi}{24}(1-\varphi t) = 0 \text{ ---------- (17)}$$

Knowing the initial and final geometry (i.e. $\Delta Lc = Lc-Lc_o$), the change of temperature and the change of vertical load, if any, a new value of "H" can be solved. This equation is of a third order that may have three roots. The correct root was found to be always larger than the largest value of "H" "e.g.H_2" which will cause either a maximum or minimum f (H) (12) & (24) – see Figs. 4&5-.

The value of "H_2" was found by solving the quadratic equation for "H" obtained by using $\partial f/\partial H= 0$ and using the positive sign before the square root quantity for the standard solution of the quadratic equation, The correct root of "H" is then found by half interval search between "H_2" and an arbitrary higher value, "H_3", i.e. (24): -

1. Set $H_4 = \frac{H_2 + H_3}{2}$, then
2. Set $H_2=H_4$ if $f(H_4)\times f(H_2) > 0$
3. Set $H_3=H_4$ if $f(H_4)\times f(H_2) < 0$
4. Repeat this process till "H_4" is elevated, then.
5. $H= H_{correct} =H_4$.

After this long operation and having solved the value of "H", the value of "ΔH" can be easily determined:

$\Delta H = H - H_o$ ------ (18)

Then the modified "E" value is: -

$$\overline{E} = E_{\text{mod.}} = \frac{\Delta H}{\Delta L_c}\frac{L_{c_o}}{A\cos\varphi} \text{ -------(19)}$$

The principle of this method for solving f (H) is shown in figure (4), and the modified "E" value is shown in figure (5). For more exact iterational modified "E" method, (eq. 6) shall be adopted as an initial value of "Eeq." then (eq. 7) shall be adopted as the governing equation of the method of equivalent modulus of elasticity of cables in the rest of "n^{th}" iterations.

The T.S.M.of the cable element in space

Two approaches are available to analyze cable structure non-linearly (17)&(23):

a. The finite element approach.

b. The beam-column approach

In the first approach two alternatives are available. A truss element representation of cable is used with (sag=0) and an increased number of elements is used to decrease the error with "E" value equal to "Ee", hence there is no need for the equivalent "E" value or it may be analyzed using one parabolic cable element with the "E" value to be taken from (eq. 6,7) having one D.O.F/each end). In this approach, the T.S.M. of cable in local coordinates is-see Fig.6- (28): -

$$[T.S.M] = [K_T] = [K_E] + [K_G] = \frac{AE_{eq}}{L_c}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} G & -G \\ -G & G \end{bmatrix} \text{---------- (20-a)}$$

where the sub-matrices:

$$[G] = \frac{F}{L_c}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ -------- (20-b), and $E_{eq.}$ are taken from (eq. 6-7).

In the second approach, which is the approach adopted in the present work, the **T.S.M.** can be determined by the following procedure first adopted by ***Saafan***[(16)] (1970):-
The basic idea is based on the original truss element in space and since the finite deflection theory is responsible for obtaining a solution tangent to the actual non-linear force-deflection curve at any intermediate stage, then the elements of this **T.S.M.** "t_{ij}" are obtained as partial derivations of the force vector [Q] with respect to each one of the displacements. Thus: -

$$T.S.M. = \left[\frac{\partial Q}{\partial U}\right] = \begin{bmatrix} \frac{\partial Q_1}{\partial U_1} & \cdots & \frac{\partial Q_1}{\partial U_6} \\ \vdots & & \vdots \\ \frac{\partial Q_o}{\partial U_1} & \cdots & \frac{\partial Q_6}{\partial U_6} \end{bmatrix} = \begin{bmatrix} t_{11} & \cdots & t_{16} \\ \vdots & & \vdots \\ t_{61} & \cdots & t_{66} \end{bmatrix}$$

$$= \frac{E_{eq}A}{L_c}\begin{bmatrix} (l^2+\varepsilon) & lm & \ln & -(l^2+\varepsilon) & -lm & -\ln \\ ml & m^2+\varepsilon & mn & -ml & -(m^2+\varepsilon) & -mn \\ nl & nm & n^2+\varepsilon & -nl & -nm & -(n^2+\varepsilon) \\ -(l^2+\varepsilon) & -lm & -\ln & l^2+\varepsilon & lm & \ln \\ -ml & -(m^2+\varepsilon) & -mn & ml & m^2+\varepsilon & mn \\ -nl & -nm & -(n^2+\varepsilon) & nl & nm & n^2+\varepsilon \end{bmatrix}(1-\varepsilon) \text{-----(21)}$$

$$[T] = \begin{bmatrix} l^2 & lm & \ln & -l^2 & -lm & -\ln \\ ml & m^2 & mn & -ml & -m^2 & -mn \\ nl & nm & n^2 & -nl & -nm & -n^2 \\ -l^2 & -lm & -\ln & l^2 & lm & \ln \\ -ml & -m^2 & -mn & ml & m^2 & nm \\ -nl & -nm & -n^2 & nl & nm & n^2 \end{bmatrix} \text{----------22}$$

So, (eq. 22), is the final T.S.M. in global coordinates adopted in the work concerning the cable element in space which is the most efficient method of analyzing the cable-frame interaction structures since it follows the actual shape of cable [(16)&(17)].

Conclusions

The most efficient method of analyzing the cable-frame interaction structures is the plane or space tangent stiffness matrix non-linear analysis (T.S.M.) method, since it follows the actual shape of cable consequently with the load applied. With the great benefits of computers this type of analysis is simpler and quicker than the actual catenary approach or the approximate parabola approach or even the non-linear approach.

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SYMBLES:

A: cable cross-sectional area.

d_c: the diameter of the cable…d_r: the diameter of the rope.

E: the modulus of elasticity of cable.

Ee: the straight cable modulus of elasticity = the effective modulus of elasticity of the cables.

Eeq.: the equivalent modulus of elasticity of the cables (taken from (eq. 6, 7)).

Ei: the sagged cable modulus of elasticity.

f: the sag of cable.

Fi , F_f: the initial and final cable tensions respectively.

F_o: the initial tensile axial force = Fi.

F_1: the tensile axial force after first stage of loading.

H: the horizontal component of the cable tensile force.
[K]: the stiffness matrix corresponding to the [Q]=[T]-1[N][T]
l: the horizontal chord length = Lh = Lc cos φ_o = = Lc (if cos φ = l).
L: the arc length of cable.
Lc: the chord length of cable.
l, m, n: the direction cosines of the member center line.
n_r: number of ropes the cable consisting of .
[N]: the stiffness matrix of the truss element $= E_e A / L_c \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
{p}: the column vector of the forces in the member axis =[N]{u}.
Q=[T]-1[P]=[T]-1[N][T][U]=[K][U] = the stress resultant related to the number of loads {P}.
[T]: the orthogonal transformation matrix.
$[T]^{-1}$: the orthogonal matrix inverse.
{u}: the column vector of the displacements in member axis
[U]: the column vector of the displacement in the common axis.
w: the load intensity (weight of the cable (force/length).
σ: Tensile stress in the cable = F/A…F: tensile axial force.
γ: specific gravity of the cable….γ_f: Δl -Δl1/ Lc.
Δ_l: L - Lc …Δl1 = L - Lc1…
φ = Cable inclination….φ_0 = initial cable inclination.
ε = the axial strain = $\left(\Delta_e \frac{L_c}{L} - \Delta_{temp.} \right)$, Δ_e = (eq. 14), $\Delta_{temp.}$=(eq.25)

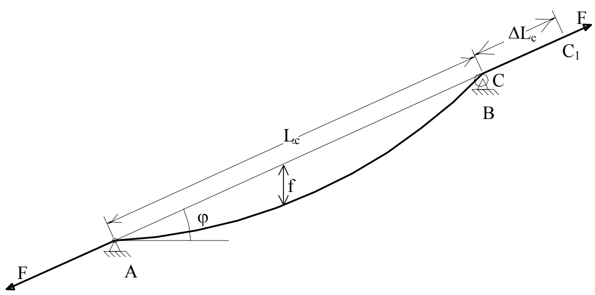


Figure (1): The inclined cable arrangement

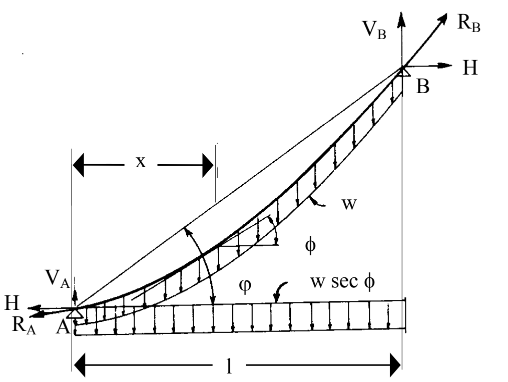


Figure (2): Inclined cable under its own weight

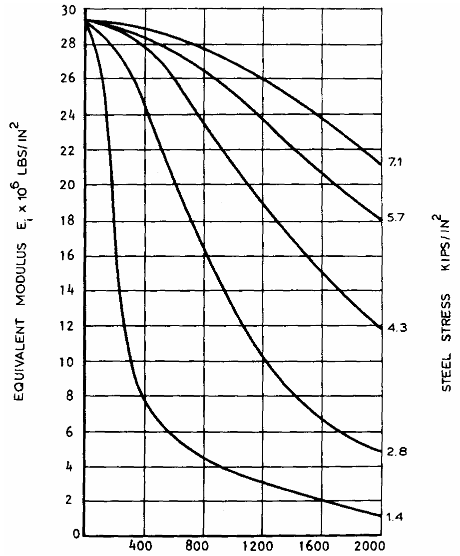


Figure (3): Variation of Ideal Modulus With Span Length

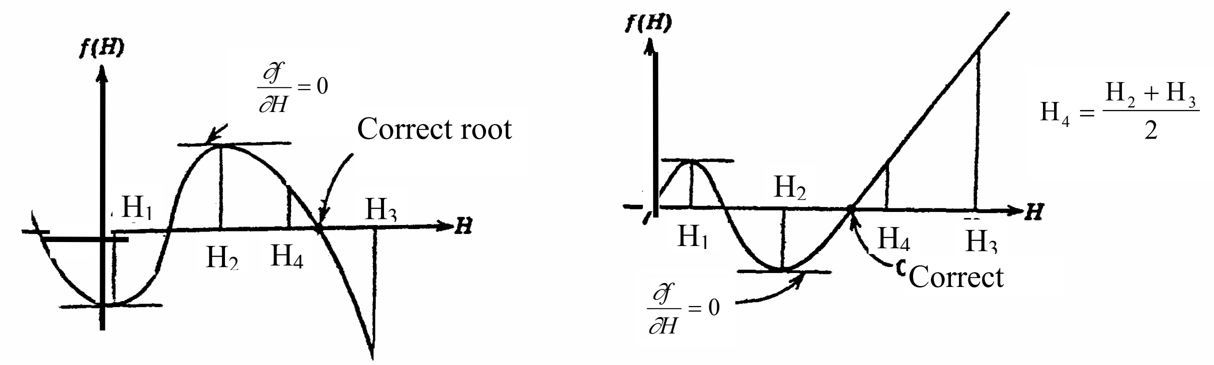


Figure (4): Method of finding correct root of cable equation.

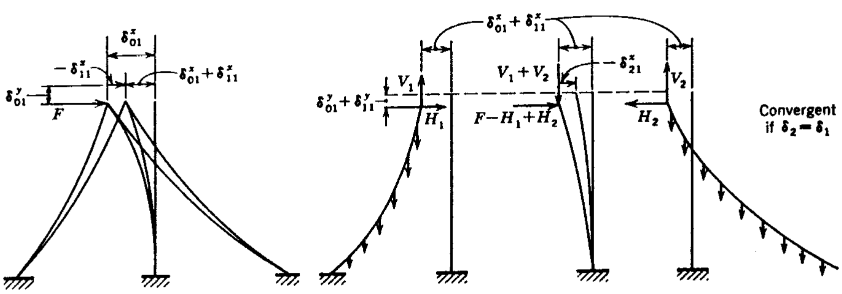


Figure (5): Principal of iterational technique in cable structure

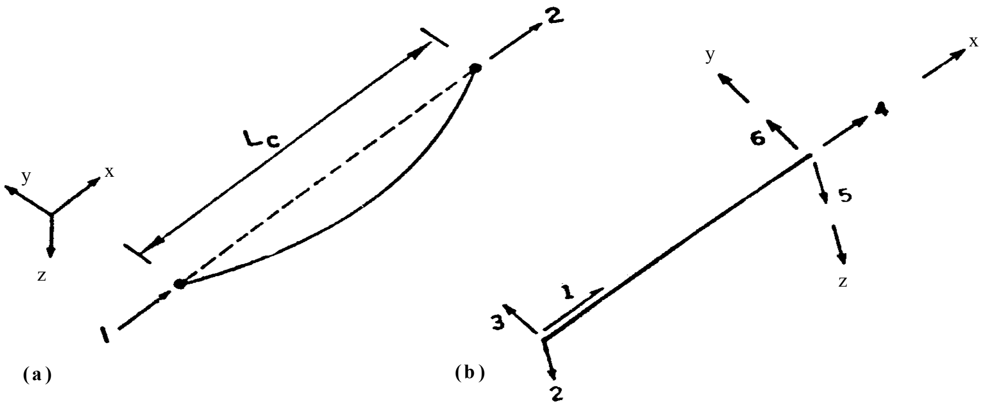


Figure (6): Typical cable element in local co-ordinates (a) actual element with sag≠0, (b) idealization and D.O.F [17].

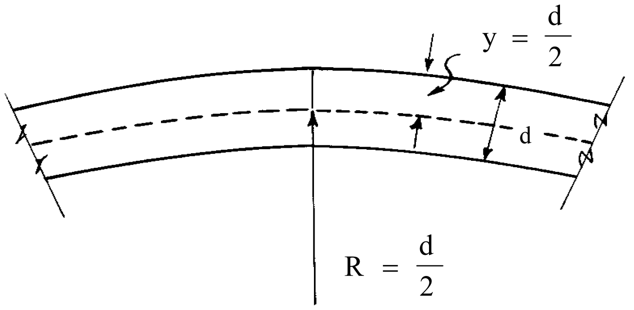
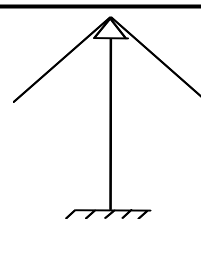
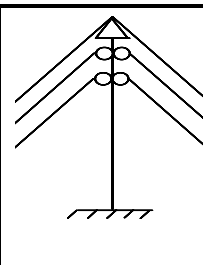
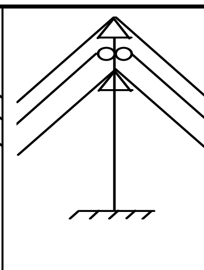
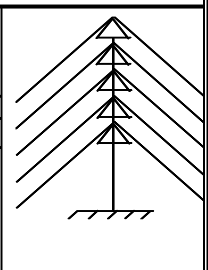
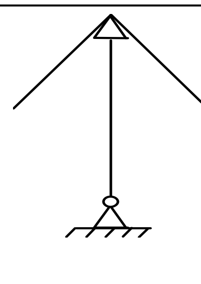
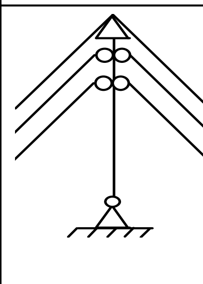
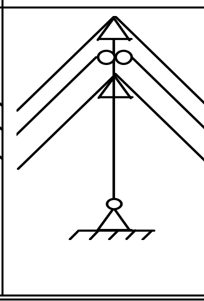
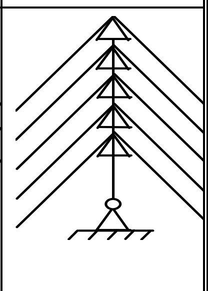


Figure (7): The negligible bending stress of wire.

Table (1): Types of cable - tower interaction

BASE	LEGEND: FIXED SADDLE ROLLER OR ROCKER SADDLE			
FIXED				
PIN-BEARING				

APPENDIX A: Types, physical properties, and behavior of cables:

A cable may be composed of one or more structural ropes, structural strands, locked coil strands or parallel wire strands.

A **strand**, with the exception of a parallel wire strand, is an assembly of wires formed helically around a center wire in one or more symmetrical layers and is produced in accordance with the ***ASTM standards A-586 specifications*** [(1)]. A strand may be used either as an individual load-carrying member, where the radius of curvature is not a major requirement, or as a component in the manufacture of structural rope. The structural strand is generally preferred to rope for cable-stayed bridges, and towers, while ropes are suitable in cable-suspended bridges and highly sagged towers. The three main types of strand configuration are the helically wound strand, the parallel wire strand, and the locked coil strand

A **rope** is composed of a plurality of strands helically laid around a core and is covered by***ASTMA -603 standard specifications*** [(1)], in contrast to the strand, a rope provides increased curvature capability and is used where curvature of the cable becomes an important consideration.

The modulus of elasticity is constant but is a function of the force acting on the cable. The rope of twisted strands that are a further step towards economy in the construction of inclined cable bridges and towers. The value of modulus of elasticity for these cables closely approaches that of the plain wire, and this fact has a favorable influence on the deformations because of the live load. The reduction of the value of the modulus of elasticity due to cable sag, however, produces the same percentage of figures as for the locked coil cables. But as there are no plastic elongations, as in cables, the erection deformations can be accurately calculated in advance. It will be therefore possible to allow for the fatigue strength between 28500 psi and 42600 psi (19.65×10^4 and 29.37×10^4 kN/m^2) provided that anchorages and curves do not impose any reduction.

The majority of the existing cable-stayed bridges use **pre-formed, pre-stretched,** or **galvanized locked-coil wire rope.** This type exhibits more effective protection against corrosion as well as more favorable properties compared with those of a conventional spiral rope manufactured from round wires, so it introduces higher density of the material with higher modulus of elasticity that is almost exactly halfway between the value of the spiral cable and the solid structural steel. The flexibility required at curves is maintained in spite of the above properties because of its spiral construction, moreover, locked-coil cables are largely insensitive to bearing pressure because the individual layers of profile wires support each other through their face and not just by point contact as in spiral cables [(21)].

The total elongation or stretch of a structural strand is the result of several component deformations such as those due to its own weight, temperature, and industrial imperfections, however, the pre-stretching meant here is the one called constructional stretch which is caused by the lengthening of the strand lay (pitch length of the wire helix) due to subsequent adjustment of the wires in a strand into a denser cross section under load, this type of stretch is permanent. According to ***ASTM A-586*** the total pre-stretch load should not exceed 55% of the rated breaking strength of the strand [(21)].

According to ***ASTM A-586*** specifications, the elongation readings for computing the modulus of elastic, they are taken when the strand or rope is stressed to not less than 10% of the minimum rated breaking strength or more than 90% of the pre-stretching load. The modulus of elasticity shall not be less than 24×10^6 psi (16.59×10^7 kN/m^2) for 1/2 - 29/16 in (12.7-65.1 mm) nominal diameter strand and 23×10^6 psi (15.86×10^7 kN/m^2) for 25/8 in (66.6 mm) and larger nominal diameter strands, these values are for normal pre-stretched helical types strands. For a parallel-wire strand the modulus of elasticity is in the range of 28×10^6 - 28.5×10^6 psi (19.30×10^7 - 19.65×10^7 kN/m^2).

The modulus of elasticity of the rope is low for low loads and increases as the load is increased into the normal working range. Creep may occur for sustained loads. The stiffness of the cable-frame structure depends largely upon the tensile stiffness of the stay cables, the apparent modulus of elasticity "E_f" may be expressed as: -

$$E_f = \sigma / \varepsilon_f \text{ ----------(A-1)}$$

While the ideal modulus of elasticity of the cable "EI" under an axial load with its own weight to be considered and a sag "f" is introduced: -

$$E_i = \frac{E_f E_e}{E_f + E_e} \text{----------(A-2)}$$

To find "Δ_l" and "Ei", one of the analytical approaches must be adopted: -

a. The catenary behavior……b. The parabolic behavior.

The comparison between the catenary and parabola indicates negligible difference [22]. Therefore, the catenary may be satisfactorily approximated over this length by a parabola. So, eq. (A-2) becomes after some derivational effort: -

$$E_i = \frac{E_e}{1 + \left[(\gamma L_h)^2 / 12\sigma^3\right] E_e} \text{ -------(A-3)}$$

If we assume a straight locked-coil steel wire rope eq. (A-3) becomes: -

$$E_i = \frac{10800}{1 + (2.42\times10^{-3}) L_h^2 / \sigma^3} \quad (\text{ton/in}^2)\text{----------(A-4)}$$, "L" in ft. and "σ" is in ton/in^2

Assuming numerical values for span "L" and stresses "σ", M.S. Trotisky [21] formulated eq. (A-4) to illustrate the (Ei-L-σ) cable behavior, this behavior is shown in figure (2).

The general factor of safety of the cable may be taken as $K_1 = 2.5$ following practice in USA and Europe. This coefficient represents the reserve of the strength of the cable with respect to the loading. The calculated strength of the cable may be expressed as: -

$$R = R_{a11} \cdot K \cdot m_1 m_2 \text{ ----- (A-5)}$$

Where $m_1 = 0.8$ is the coefficient of the performance of material in structure.

$$m_2 = \frac{1}{K_{m_1} K_{res}} = 0.78$$

Some authors take it as 0.74), Kres ≈ 2.

Flexural and shear behavior of bent cable: Under longitudinal tension applied to a spirally wound rope, transverse forces are developed between the individual wires. This reduces the ultimate capacity below the sum of the capacities of the individual wires for two reasons: -

a. A combined stress system exists at the contact areas: plastic flow at the contact areas causes necking of the wires and results in reduction in their cross-sectional areas.

b. When rope is bent over a saddle, two additional effects are present: -

c. Bending stresses are set up, and

d. Transverse radial forces are applied to the rope.

For single wire of diameter "d" bent to a diameter "D" at its centerline, the bending stress is calculated (see figure.7).

- After substituting into the formula f = My/I the value M = EI/R, we have:

$$f = \mathrm{E}\frac{y}{R} = \mathrm{E}\frac{d}{D}$$ ---------- (A-6), Where "E" is the modulus of elasticity of the wire.

For a spirally wound rope, over a saddle of a moderate length, it appears probable that shear movement can occur between adjacent wires, and that the bending stress approaches the value given in (eq.A- 6), with "d" equal to the diameter of the rope.

Considering that each wire in the strand makes one complete turn around the core of a strand, each wire takes either the upper or lower position and the corresponding stresses change their signs. This results in the equalizing of the stresses in the wires; in addition, because of the friction effect under transverse compression, there is a certain redistribution of the stresses among the separate wires in the cross-section of the cable. Because of these factors the bending of the cables over a saddle is disregarded in the analysis and the cable is not represented as a beam-column element, so the flexural rigidity and the shear rigidity are ignored. However the "I" and the "u" values shall be calculated and included in the case studies of this work to illustrate how small they are.

In spite of a small value of the shear parameter "u" of the cable, there is a small notation here. It must be considered that the shape factor "n" of cable cannot be calculated as ($n = 1.11$) for the corresponding solid circular section but the cable cross section consists of a number of ropes and wires and since the actual shape factor definition is: -

$$\mathrm{n} = \frac{\mathrm{A}}{\mathrm{A_w}} = \frac{d_c}{n_r d_r^2}$$ ---------- (A-8)

Although, the "n" value makes no sense since the small effect of shear in the contribution of the cable element stiffness but it obviously differs from the (1.11) value of (eq.A-8), again this equation has originality. More important is the effect of the transverse compression of the cables, which results in bi-axial stress of different sign in the wires, which leads to reduced bearing capacity of the material (21). The above argument means that there is no bowing effect to be added to the axial displacement. The torsional moment however dose affect the axial displacement but the technique used in the cable-tower interaction joints does not permit the cable to twist since it transmits the twisting moments applied on the cables due to joint movements of rockers connected to the point of interaction between cable and mast.